Mildenhall Ch 3: Risk and Risk Measures

1.1 Introduction

The term **Risk** refers to the uncertainty of achieving objectives

- Note that insurers can also use the term "risk" to refer to a peril or a account

The text distinguishes between various types of risk:

- **Pure risk/ Insurance risk**: possible bad outcome, with no potential of a good outcome
  - The loss on an insurance policy is an example of a pure risk
- **Speculative risk/ Asset risk**: this could have a good or bad outcome
  - If we instead focus on the net position of insurance (premium − loss & expense) as opposed to just the loss, then insurance can be considered to be a speculative risk

Financial risk refers to the uncertainty of financial outcomes.

- Financial risk can involve uncertainty in timing, amount, or both

Insurance can reduce this financial risk to the insured by specifying payment dates, or covering a portion of the loss.

1.2 Diversifiable Risk

Insurance benefits from the concept of **diversification**, where adding an independent policy to a portfolio increases the aggregate risk by less than the risk of the added policy on a standalone basis.

- The insurer needs to understand if the added risk is diversifiable (aka **idiosyncratic**)
- Risk that is nondiversifiable is said to be **systematic**. This typically means that:
  - there is a common underlying cause or some other factor causing dependence among multiple units (e.g. catastrophes that impact multiple units), or
  - there is a single unit that has a large influence on the total loss (e.g. a line of business that has huge losses relative to the other lines)

Note that systematic risk is different to **systemic risk**, which is where an individual event can cause a chain reaction of additional events. The financial market is typically impacted by systemic risk as it consists of many interacting parties, and therefore a single event can impact many of the participants.

- Regulators have identified **Systemically important financial institutions** (SIFIs), which are firms that are considered to generate systemic risk, and are considered to be "too big to fail"

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- Even though P&C insurers are not typically considered to be SIFIs (because they have a combination of a large amounts of liquid assets and illiquid liabilities), certain insurers were classified as SIFIs following the Global Financial Crisis of 2008

Catastrophes are a good example to illustrate the difference between systematic and systemic risk:
- Catastrophes are considered to be systematic risk, since many insureds are impacted.
- Catastrophes are not considered to be systemic, as in general, losses are not amplified by a chain reaction of events.

1.3 Types of Uncertainty

The text compares Objective and Subjective probabilities:
- **Objective probability**: probabilities can be calculated precisely. Insurance is usually based objective probabilities that can be derived from loss data.
- **Subjective probability**: judgment is usually required to estimate this subjective probability. Used for nonrepeatable events, such as the results of a horse race. Insurance tries to avoid pricing based on subjective probabilities as this is much more difficult to do accurately (e.g. pricing for terrorism risk)

It also compares process risk and parameter risk:
- **Process risk**: the uncertainty that arises from a random process
- **Parameter risk**: the uncertainty in estimating the parameters

1.4 Representing Risky Outcomes

There are several different ways to label a risky outcome:
- Explicit: describe the factors that cause it
- Implicit: identify the outcome by its value
- Dual implicit: identify it by the probability of observing no larger value

These are all described below.

1.4.1 Explicit Representation

An **explicit representation** identifies an event by providing specific details about it. For example, the following factors can be used for an auto loss:
- Policy number
- VIN
- Date & time of loss
- GPS location of accident

Explicit representation can distinguish between different events based on these factors, even if they have the same loss dollars.

Let Ω represent the universe of different combinations of selected variables (the **"Sample space"**). Each specific combination within the sample space is represented as $\omega \in \Omega$.

Note that there may be other variables in the dataset that are not vital to identify the event (for example, loss adjuster name). These ancillary fields are not included in ω, but rather are considered to be "functions" of ω, and can be linked to the event if necessary.

Lloyd's of London uses Explicit representation to aggregate catastrophe risk: each member syndicate needs to report estimated losses from 16 "Realistic Disaster Scenarios" (RDS), including Miami Windstorm and Japan Typhoon, and these losses are aggregated. *This will be discussed further in the text.*

The main advantage of this approach is the ability to link outcomes across a book of business, which helps in the modeling of dependence risk. However, if there are too many possible events, or if the events impact only a small portion of the portfolio, it likely does not make sense to use explicit representation as there isn't sufficient benefit relative to the complexity of applying this practice.

1.5 Implicit Representation

An **implicit representation** defines an event by its value. This may be appropriate to use if the loss outcome is what is important, as opposed to the cause of loss.

The Sample space in this case is any value: $(-\infty, \infty)$

There are a few disadvantages of this approach:

- It is difficult to aggregate events as there is no way to link the different outcomes of an event
- It is difficult to specify dependence
- We can not distinguish between different events that have the same outcome

As an example, assume that the cat model output for an insurer covers both the homeowners and commercial lines of business. The spreadsheet with the output for Homeowners has the following columns:

- Hurricane/ Earthquake flag
- Homeowners loss

This data can be used to derive Homeowners loss distributions by peril as well as in total. However it can not be used to link a Homeowners loss to a Commercial loss from the same event, as we only have the value and therefore there is no way to connect the two.

1.6 Dual Implicit Representation

Dual implicit representation is even more of a simplification than Implicit representation. It could be used if we don't care about the loss size, but rather just the rank of the outcomes.

An outcome $X = x$ can be identified by its nonexceedance probability:

$$p = F(x) = Pr(X \leq x)$$

An equivalent way to look at this is with the exceedance probability:

$$s = S(x) = Pr(X > x)$$

The Sample space in this case is: (0, 1)

We can also determine x from a given p value by using the inverse of the distribution function (excel has functions to accomplish this such as BETA.INV, GAMMA.INV, LONGNORM.INV, NORM.INV, etc).

Example:
Assume that a normal distribution has a mean of 0 and SD of 1. If the p value is 0.7, calculate x

In Excel, enter NORM.INV(0.7,0,1)
x = 0.524

Examples of dual implicit representation include:

- Investors assess bonds based on their estimated probability of default
- Results of catastrophe models are often summarized by the exceedance probability

One of the advantages of this representation is that it is easy to make comparisons (especially of different events), as regardless of the value of X, F(X) always lies between 0 and 1.

Disadvantages include:

- Similar to implicit representation, it is difficult to aggregate
  - Rating agencies and regulators using this approach would be less concerned with this disadvantage, as they are focused on the standalone results of the insurer.
- In practice, events are often compared to an unspecified reference portfolio. For example some news reports described Hurricane Katrina as a 1 in 25 year event (which was comparing to all hurricanes landing in the U.S.), whereas other reports described it as a 1 in 300 year event (comparing to storms that were in the same area).

1.7 Conclusion on Representations

The representations discussed previously are all used in insurance. For example:

- Explicit: specific claim file
- Implicit: loss amount from the claim (ignores other details)
- Dual implicit: nonexceedance probability of the loss

Risk Measures

2.1 Intro

A **risk preference** is a way of measuring preferences between different risks.

- X $\succeq$ Y: risk X is preferred to Y
- If both X $\succeq$ Y and Y $\succeq$ X: we are neutral between X & Y

A risk preference for insurance loss outcomes needs to have the following properties:

- **Complete (COM)**: for any pair of losses X & Y, we can conclude that X $\succeq$ Y, or Y $\succeq$ X, or both
  - In this case, we are able to perform comparisons between any two pairs
- **Transitive (TR)**: If X $\succeq$ Y and Y $\succeq$ Z, then X $\succeq$ Z
  - This means that the risk preference is logically consistent
- **Monotonic (MONO)**: If X $\leq$ Y for all outcomes, then X $\succeq$ Y

A **risk measure** is a way to quantify risk preferences (via a single number), which facilitates decision making. RBC is an example of a risk measure. If ρ is the risk measure:

$$X \succeq Y \Leftrightarrow \rho(X) \leq \rho(Y)$$

The following factors impact risk measures:

- **Volume**: smaller risks are preferred
- **Volatility**: lower volatility is preferred
- **Tail**: lower tail is preferred
  - This is not necessarily the same as volatility. For example, the chance of winning \$1M or \$3M has the same variability as the chance of winning \$1M or losing \$1M; but the tail risk in the latter case is much more severe

2.2 Applications of Risk Measures

Insurance company operations are heavily driven by the following risk measures:

- **Capital risk measure**:
  - generates the capital need to write the desired business
  - based on the amount of assets required to support the insurer at a given level of confidence
  - an alternative way to approach this is to determine the business that the insurer can write based on a hypothetical capital amount
  - the capital risk measure is typically more sensitive to tail risk as capital is typically required to support tail events that threaten the insurer's solvency
- **Pricing risk measure**:
  - determines the cost of insurance required to compensate the investors for bearing the risk
  - the resulting **margin** (equal to premium − expected loss) needs to be sufficient for the insurer to be able to attract the necessary capital (from investors)
  - this measure is typically more sensitive to volatility as management is concerned about volatility of earnings in addition to solvency

The risk measures have several uses. The text briefly discusses the following two, which are related to **conservatism**:

- Treat the level of conservatism as an input, and derive the required premium or capital based on this
  - e.g. generate the capital needed so that the insurer is able to pay for 99% of the loss outcomes
- Use the actual price or capital requirement of the insurer to back into the implied level of conservatism
  - e.g. determine the percentage of loss outcomes that the insurer is able to pay based on the available capital

Stress tests can also be used as the risk measure. For example, Llyods uses the Realistic Disaster Scenarios (described elsewhere) as a risk measure to set capital requirements. It could set capital equal to the worse outcome. Since this is fairly intuitive, it helps communicate its capacity to a nontechnical audience.

2.3 Risk Measure Functional Forms

There are many different functional forms that we can use for risk measures. One of the most basic options for a loss X is the mean of X:

$$E[X] = \int_{\Omega} X(\omega)Pr(d\omega) = \int xdF(x)$$

The authors then go on to discuss various risk measures that are different versions of the expected value:

$$\int_{\Omega} g(X(\omega), \omega)Pr(d\omega)$$

- a. Here the outcome value is adjusted by a factor that depends on both the outcome value & the sample point ω.

$$\int_{\Omega} X(\omega)Pr^*(d\omega)$$

- b. Here the probabilities are adjusted. For example, this adjustment can be used to make specific events possible or impossible.

$$\int_{\Omega} X(\omega)p(\omega)Pr(d\omega)$$

- c. Here the original probabilities are scaled. Events that were previously impossible would remain impossible under this adjustment.

$$h(\int_{\Omega} g(X(\omega))Pr(d\omega))$$

- d. Here the adjustment is actually a function of the loss as opposed to ω. Note that standard deviation has this form, where $h(x) = x^{0.5}$ and $g(x) = (x - \mu)^2$

$$\int_0^{\infty} \mu(x)dF_X(x)$$

- e. This does not adjust the probabilities, but does adjust the outcomes independently of ω.

$$\int_0^{\infty} xg'(S_X(x))dF_X(x)$$

- f. Here the probabilities are adjusted based on the rank of the loss. This leads to "Spectral Risk Measures", which are discussed later.

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$$\int_0^\infty \mu(x)g'(S_X(x))dF_X(x)$$

- g. This is a combination of the prior two approaches.

Note:

- The authors believe that it is preferable to adjust probabilities instead of outcomes
- In the final 3 methods, the result depends on the distribution of X as opposed to its magnitude. We say that risk measures with this property are **law invariant**.
- It is important that the measure(s) selected are appropriate for the intended use. The user also has the option to generate multiple different risk measures and use a weighted average of the results.
- Once the risk measure has been created, we can perform additional functions on the result. For example, we can gross up the result to account for operational and unmodeled risk (*discussed elsewhere in the syllabus*).

