

Solutions to Past Exam Problems

1. 2000 Exam 9 - Q32 (3 points)

(a)

Rating	Premium (\$000)	Claim Counts	Counts/Prem	Freq Relative to Total	Credibility
A	25,846	31,964	1.237	0.920	0.080
A+X	27,629	34,659	1.254	0.933	0.067
A+X+Y	29,910	38,205	1.277	0.950	0.050
Total	34,039	45,770	1.345	1	

(b) The credibilities should increase in proportion to the # of years of experience if the chance of accidents for individual risks remains constant and no risks enter or leave the class.

(c) 2 Years relative to 1 year: $0.067 / 0.050 = 1.338$
 3 Years relative to 1 year: $0.080 / 0.050 = 1.590$

Since these are much less than 2 and 3, respectively, it must be that risks' chances for accidents are changing and/or risks may be entering or leaving the class.

2. 2001 Exam 9 - Q2 revised (1 point)

- (a) True: this is one of the main points of the Bailey & Simon paper.
- (b) True: the more difference between individual risks in a class, the more powerful individual risk rating will become.
- (c) True: if the variance in loss experience between risks is largely explained by the classification rating variables, then experience rating wouldn't add much predictive power.
- (d) FALSE: This is in contrast to Bailey & Simon's 3rd conclusion.

3. 2001 Exam 9 - Q22 (2.5 points)

- (a) I assume the problem meant 1 or more years and 2 or more years claim-free, since we are not given data for exactly 2 years.

I also assume the premium is at present 0 years claim-free rates.

Note that table of information all reflects current year data, which we will use to determine the Mod. We don't have prior year data, but we know R will equal 0 for both 1+ and 2+ years claim-free.

$$1+ \text{ years Mod} = \frac{(7,000+10,000)/(\$5,000,000+\$7,000,000)}{19,000/\$13,000,000} = 0.969$$

$$\text{Since } R = 0 \text{ for } 1+ \text{ years claim-free, } Z = 1 - \text{Mod} = 1 - 0.969 = 0.031$$

$$2+ \text{ years Mod} = \frac{7,000/\$5,000,000}{19,000/\$13,000,000} = 0.958$$

$$\text{Since } R = 0 \text{ for } 2+ \text{ years claim-free, } Z = 1 - \text{Mod} = 1 - 0.958 = 0.042$$

- (b) The authors use earned premium at current class B rates as their exposure base to avoid the maldistribution caused when higher frequency territories produce more X, Y, and B risks and higher premiums.
- (c)
- High frequency territories are also high average premium territories.
 - Territorial rate differentials are proper.

4. 2002 Exam 9 - Q47 (2 points)

Note: the terminology here for "class" is different than in the source paper. In the source paper, all of the merit ratings have different factors, but can be contained within a single broader class.

- (a) **Solution assuming 1+ and 2+ years claim-free (more in line with the paper and what the CAS intended):**

I assume the problem meant 1+ and 2+ years claim-free, and not exactly 1 and 2 years claim-free.

$$A+X+Y \text{ Mod} = \frac{(850+1,000+5,000)/(\$535,000+\$682,500+\$5,500,000)}{7,750/\$7,208,000} = 0.948$$

$$A+X+Y \text{ Credibility} = 1 - 0.948 = 0.052$$

$$A+X \text{ Mod} = \frac{(1,000+5,000)/(\$682,500+\$5,500,000)}{7,750/\$7,208,000} = 0.903$$

$$A+X \text{ Credibility} = 1 - 0.903 = 0.097$$

Solution for EXACTLY 1 and 2 years claim-free:

$$Y \text{ Mod} = \frac{850/\$535,000}{7,750/\$7,208,000} = 1.478$$

$$Y \text{ Credibility} = 1 - 1.478 = -0.478$$

$$X \text{ Mod} = \frac{1,000/\$682,500}{7,750/\$7,208,000} = 1.363$$

$$X \text{ Credibility} = 1 - 1.363 = -0.363$$

Note that the credibilities are negative. The reason for this is that the Mod is applied to the class rate, and the class mostly consists of the 3+ years claims-free insureds. As such, if the class rate is set to be the weighted average loss cost for all insureds in the class, the class rate will be low, and the factors for X and Y ratings will be higher than the class rate. So for X and Y insureds, the Mod > 1, and R = 0 since they were claim-free last year, thus a negative credibility results.

- (b)
- Risks are entering/exiting the portfolio.
 - Risk characteristics are changing over time.

5. 2003 Exam 9 - Q2 (1 point)

- (a) FALSE: the more refined the classification plan, the less need for experience rating. In other words, if all the variation in losses is explained through rating variables, then using an individual risks's past experience adds no value.
- (b) True: The more variation within the class, the more meaningful experience rating will be, since classification rating does not sufficiently explain the variation in losses between risks.
- (c) True: This is a conclusion of the Bailey & Simon paper.
- (d) True: This is basically the same as (b) above.

6. 2003 Exam 9 - Q22 (3 points)

Note: the terminology here for "class" is different than in the source paper. In the source paper, all of the merit ratings have different factors, but can be contained within a single broader class.

$$(a) \text{ Average Frequency} = \frac{200+12+20+38}{375,000+15,000+22,500+37,500} = 0.0006$$

$$\text{Class A Frequency} = 200/375,000 = 0.000533$$

$$\text{Mod} = 0.000533/0.0006 = 0.8889$$

$$Z_A = 1 - 0.8889 = 0.1111$$

$$(b) \text{ Class A + X + Y Frequency} = \frac{200+12+20}{375,000+15,000+22,500} = 0.00056$$

$$\text{Mod} = 0.00056/0.0006 = 0.9374$$

$$Z_{A+X+Y} = 1 - 0.9374 = 0.0626$$

- (c)
- Risks are entering/exiting the portfolio.
 - Risk characteristics are changing over time.

7. 2004 Exam 9 - Q2 revised (1 point)

Note: the terminology here for "class" is different than in the source paper. In the source paper, all of the merit ratings have different factors, but can be contained within a single broader class.

$$\text{Mod} = \frac{(1,250+1,155+1,000) / (\$625,000+\$770,000+\$1,000,000)}{4,405 / \$2,795,000} = 0.902$$

$$Z = 1 - 0.902 = 0.098$$

8. 2005 Exam 9 - Q3 (3 points)(a) Let $X = \#$ of claims

$$\Pr(\text{risk was accident free last year}) = \Pr(X = 0) = e^{-m}$$

$$\Pr(\text{risk had at least one accident last year}) = 1 - \Pr(X = 0) = 1 - e^{-m}$$

$$\# \text{ of risks accident free last year} = N \times \Pr(X = 0) = N(e^{-m})$$

$$\# \text{ of risks with at least one accident last year; } N \times \Pr(X > 0) = N(1 - e^{-m})$$

$$\text{expected \# of claims last year} = Nm$$

$$\text{freq} = \text{avg \# of claims last year for current B risks} = (Nm) / [N(1 - e^{-m})] = m / (1 - e^{-m})$$

$$R = (\text{freq for B}) / (\text{overall frequency}) = (m / (1 - e^{-m})) / m = (1 / (1 - e^{-m}))$$

$$Z = (M - 1) / (R - 1) = (M - 1) / ((1 / (1 - e^{-m})) - 1)$$

(b) Credibility for an individual risk is lowered when the class plan is highly refined, because it is more difficult to identify differences in the loss potential for the particular risk at-hand from the average risk in the class.

9. 2006 Exam 9 - Q2 (4 points)

(a) Solution assuming 1+, 2+, and 3+ years claim-free (more in line with the paper):

I assume the problem meant 1+, 2+, and 3+ years claim-free, and not exactly 1, 2, and 3 years claim-free.

Group	EP(000)	Claims	Freq	Mod = Freq / Total Freq	Cred = 1-Mod
3+	25,000	40,000	40/25 = 1.6	0.785	21.5%
2+	33,000	55,000	55/33 = 1.667	0.818	18.2%
1+	46,000	80,000	80/46 = 1.739	0.854	14.6%
Total	54,000	110,000	110/54 = 2.037		

Solution for EXACTLY 1, 2, and 3 years claim-free:

$$\text{Total Frequency} = \frac{40,000 + 15,000 + 25,000 + 30,000}{\$25,000,000 + \$8,000,000 + \$13,000,000 + \$8,000,000} = 0.00204$$

$$1 \text{ Year Mod} = \frac{25,000 / \$13,000,000}{0.00204} = 0.944$$

$$1 \text{ Year Credibility} = 1 - 0.944 = 0.056$$

$$2 \text{ Year Mod} = \frac{15,000 / \$8,000,000}{0.00204} = 0.920$$

$$2 \text{ Year Credibility} = 1 - 0.920 = 0.080$$

$$3 \text{ Year Mod} = \frac{40,000 / \$25,000,000}{0.00204} = 0.785$$

$$3 \text{ Year Credibility} = 1 - 0.785 = 0.215$$

- (b) Using premium is preferable as it will account for any exposure correlation with other variables like territory.

10. 2007 Exam 9 - Q2 (3.5 points)

- (a) 1+ years frequency = $(120 + 25 + 44) / (100 + 10 + 17) = .149\%$
 2+ years frequency = $(120 + 25) / (100 + 10) = .132\%$
 3+ years frequency = $120 / 100 = .12\%$
 Overall frequency = $225,000 / 137M = .164\%$

- i 1 or more Z = $1 - .149 / .164 = 9.1\%$
 ii 2 or more Z = $1 - .132 / .164 = 19.5\%$
 iii 3 or more Z = $1 - .12 / .164 = 26.8\%$

- (b) *The declining credibility for increased years of experience for XYZ is unusual, but not theoretically impossible. The focus here is on relative credibilities and magnitude of the credibilities.*

Relative credibilities for ABC are nearly 1:2:3, but very different for XYZ. Insurer XYZ may have more risk entering and leaving classes than ABC.

You could have instead suggested risks characteristics are changing over time for XYZ.

Insurer XYZ's class plan may be more refined since the resulting credibilities are lower than ABC's (assuming both portfolios have equal total frequency).

A higher overall frequency would imply higher credibilities, so this could also be a reason for the difference between books.

- (c) If one portfolio has a more refined class plan then the credibility assigned to the experience of a single car would be lower relative to the other portfolio which has a less refined plan (assuming both portfolios have equal total frequency).

11. 2008 Exam 9 - Q5 (2 points)

- (a) *Note: in this problem, we cannot use relative frequency as we would for other problems to obtain the Mod since we don't have claim counts. As such, we have to use relative pure premium.*

$$\text{Mod} = \frac{(\$500,000 + \$1,000,000) / (500 + 2,500)}{\$4,000,000 / 4,000} = 0.5$$

$$Z = 1 - 0.5 = 0.5$$

- (b) $\text{Mod} = \frac{(\$1,000,000) / 2,500}{\$4,000,000 / 4,000} = 0.4$

$$\text{Premium} = \text{Base Rate} \times \text{Mod} = (\$1,250)(0.40) = \$500$$

12. 2009 Exam 9 - Q4 (3.5 points)

$$(a) \text{ Mod} = ZR + (1 - Z)$$

$$\text{Mod} = (18,000 / 45,000) / (100,000 / 670,000) = 2.68$$

$$2.68 = 0.167R + (1 - 0.167) \implies R = 11.05988$$

$$11.05988 = 1 / (1 - e^{-\lambda}) \implies \lambda = 0.09477$$

$$0.09477 = 100,000 / (980,000 + M) \implies M = 75,198.40$$

$$(b) \text{ Mod} = \frac{(50+20)/(400+150)}{100/670} = 0.85273$$

$$Z = 1 - \text{Mod} = 0.14727$$

13. 2010 Exam 9 - Q5 (1 point)

$$\text{Mod} = \frac{(29,300+15,000+45,000)/(20M+15M+60M)}{108,000/100M} = 0.87$$

$$Z = 1 - \text{Mod} = 0.13$$

14. 2011 Exam 8 - Q1 (3 points)

This question is really asking about which state has more variation in accident probabilities OVER TIME (i.e., not the variation within each book).

State X # of yrs clm free	EP	# clms	Rel. Clm Free (M)	Z = 1 - M	n yr Z / 1 yr Z
3+	500,000	240	$\frac{240/500,000}{855/1,150,000} = 0.6456$	0.354	2.90
2+	650,000	365	$\frac{365/650,000}{855/1,150,000} = 0.755$	0.245	2.00
1+	850,000	555	$\frac{555/850,000}{855/1,150,000} = 0.878$	0.122	1.00
0	300,000	300			
	1,150,000	855			

State Y	Mod	Z = 1 - M	n yr Z / 1 yr Z
3+	0.70	0.30	1.875
2+	0.77	0.23	1.438
1+	0.84	0.16	1.00

State X's n yr Z / 1 yr Z ratio is closer to 3, 2, 1 for 3+, 2+, 1+

- State X is more stable over time
- State Y has more variation over time

15. 2012 Exam 8 - Q6 (2.5 points)

The graders were looking for a solution here based on the text, so they wanted you to check how frequency correlates with premiums and whether loss ratios were flat. Based on that, they wanted you to conclude that you should use car-years as the base. That said, so long as territory relativities are proper, it would be fine to use premium as the base, though the graders didn't give credit for that.

Premium should be used as the exposure base to prevent the maldistribution of premium if higher frequency territories have higher premiums and territory relativities are proper. Testing this with the data shows:

Territory	Frequency ($\sum \text{claims} / \sum \text{car years}$)	avg Prem ($\sum \text{Prem} / \sum \text{car years}$)	Loss Ratio
1	0.330	1,000	0.6
2	0.277	1,029	0.6
3	0.354	893	0.6

All territories have the same Loss Ratio, which suggests the territory relativities are proper. However, higher frequency territories do not have higher average premiums. Therefore, it is advisable to use earned car years as the exposure base instead of earned premium.

16. 2014 Exam 8 - Q5 (2.5 points)

There is an issue with the solution in the examiner's report for this problem. The question itself isn't inherently flawed, so it isn't a defective question (that said, it is a bit unusual to see a higher average premium and higher frequency for people with 3+ years accident free compared to only 2 years accident free).

Using earned premium as the exposure base is basically akin to using the loss ratio method versus the pure premium method to price relativities. When there is only 1 rating variable in the rating algorithm, there will be no exposure correlation between rating variables, and both approaches would give identical results. In other words, in this problem, since the merit plan is the only rating variable, using exposures or premium should give you the exact same answer if done correctly. So the graders should have given full credit to using exposures in part (b) and using the Mod calculated based on exposures in part (c).

You could have also solved this problem using premium, but not in the way shown in the examiners report, and it would have taken longer than using exposures. If you were to use premium, the premium needs to be at a common level (I assume it is already at the current rate level). You can do this by first calculating the current relativities by dividing the premiums by the base rate of \$1,000 and then by the exposures. You'll get relativities of 1, 0.33, 4, and 12.50 for 3+, 2, 1, and 0 years, respectively. Then you can divide the premium numbers by these relativities to obtain premiums at the common level of 3+ years claims-free. Then you can use these premiums at the common level to calculate frequencies, and you'll end up with the exact same answers to parts (b) and (c) as if you had used exposures as the base instead of premiums.

- (a) i. High frequency territories are also high average premium territories.
ii. Territorial differentials are proper.

$$(b) \begin{aligned} 2+ \text{ years frequency (to ECY)} &= (1,200 + 625) / (250,000 + 300,000) = 0.0033 \\ 1+ \text{ years frequency (to ECY)} &= (1,200 + 625 + 750) / (250,000 + 300,000 + 25,000) = 0.0045 \\ \text{Total frequency (to ECY)} &= 4,075 / 587,000 = 0.0069 \end{aligned}$$

$$2+ \text{ years Mod} = 0.0033 / 0.0069 = 0.4780$$

$$1+ \text{ years Mod} = 0.0045 / 0.0069 = 0.6451$$

$$2+ \text{ years Credibility} = 1 - 0.4780 = 0.5220$$

$$1+ \text{ years Credibility} = 1 - 0.6451 = 0.3550$$

$$\text{Ratio of 2+ to 1+ Credibility} = 0.5220 / 0.3550 = 1.471$$

$$(c) \text{ Premium} = \$1,000 \times 0.4780 = \$478.00$$

17. 2015 Exam 8 - Q1 (2.5 points)

- (a) We would want to use earned premium as an exposure base if there is exposure correlation between territory and number of years accident-free and territory relativities are properly priced. There may be correlation since frequency varies by territory, but since territories are not properly priced due to not being allowed in rating, premium will not be an improvement over earned car years as an exposure base.

To clarify, assuming merit rating is the only rating variable (since we are told territory is not used for rating and we aren't told about any other rating variables), using either exposures or premium would work equally well in this case since there would be no exposure correlation with other rating variables (as there are none with this assumption). However, if there are additional variables in the rating plan that do have exposure correlation with merit rating, then using premium would definitely be an improvement over using exposures.

- (b) $1+ \text{ yrs } R = 0$ by definition since 1+ years accident free. That means $\text{Mod} = \text{Relative Frequency} = 1 - Z$.

$$1+ \text{ yrs Freq} = 35 / 700 = 0.05$$

$$\text{Total Freq} = 44 / 800 = 0.055$$

$$1+ \text{ yrs Mod} = 0.05 / 0.055 = 0.9091$$

$$1+ \text{ yrs Cred} = 1 - 0.9091 = 0.0909$$

- (c) $0 \text{ yrs } R = 1 / (1 - e^{-0.055}) = 18.686$

$$0 \text{ yrs Freq} = 9 / 100 = 0.09$$

$$0 \text{ yrs Mod} = 0.09 / 0.055 = 1.6364$$

$$0 \text{ yrs Cred} = (1.6364 - 1) / (18.686 - 1) = 0.0360$$

18. 2016 Exam 8 - Q1 (2.75 points)

This table is clearly related to the table in Appendix 1 of the Bailey & Simon paper. However, the purpose of that table is to demonstrate that with a fixed cohort of risks with constant frequencies, you can give about twice as much individual risk credibility to a risk with 2+ years-claims free compared to a risk with 1+ years claims-free (and about 3 times the credibility for 3+ years). In other words, the lack of variation of an individual insured's chance of an accident is taken as a given in creating that table, rather than learned as a conclusion from the table. So assuming the group of insureds in this question has no one enter or leave the group, then the makeup of the group is constant, and with constant expected frequencies, an individual insured randomly chosen from the group would have the same expected frequency no matter when the insured is chosen in time (note that insureds with claims don't actually leave the group, so they would still be included in calculating the expected frequency for the group). Thus, of course the variation of an individual insured's chance for an accident won't change over time - as mentioned above, this is already assumed when you have a fixed cohort of risks with constant frequencies. It seems like the question writer didn't understand this point, and as such, I believe you'll have to essentially work backwards by seeing whether the 2+ and 3+ year credibilities are 2 and 3 times the 1+ year credibility, and if so, only then do you conclude that the variation of an individual insured's chance of an accident is not changing over time. Given this, I'll show the solution that is consistent with the source paper, and then 1 more based on an alternative interpretation of the numbers given in the problem that is not consistent with the source paper (not sure if the graders will allow it).

SOLUTION 1: Source paper approach

We can interpret the given table such that $t=1$ means 1+ years claim-free, $t=2$ means 2+ years claim-free, and $t=3$ means 3+ years claim-free. In other words, if an insured has made it to time 1 without claims, then that insured might also make it to times 2 or 3 or so on without claims. So really, the numbers given are not EXACTLY t years claim-free, but t or more years claim-free. So if we wanted the number of policies with EXACTLY t years claims-free, we can subtract adjacent columns in the given table. For the 0.05 freq group, this would mean 2,500 insured have exactly 0 years claims-free, 2,500 have exactly 1 year claims-free, 1,000 insureds have exactly 2 years claims-free, and 44,000 insureds have 3+ years claims-free.

of claims at t = # of insureds claim-free at t × Expected Freq
 Freq at t = total # of claims at t / total # of insureds claim-free at t
 Credibility = 1 - Relative Freq to $t=0$ (note that $t=0$ is the total of all insureds)

Group	# of claims from			
	t=0	t=1	t=2	t=3
0.05	2,500	2,375	2,250	2,200
0.10	5,000	4,500	4,300	3,600
0.20	5,000	4,100	3,300	2,800
Total	12,500	10,975	9,850	8,600
Freq	0.1000	0.0971	0.0943	0.0915
Relative Freq to t=0	1	0.9712	0.9426	0.9149
Credibility		0.0288	0.0574	0.0851
Relative Cred to t=0		1	1.996	2.959

Since the relative credibilities for $t=2$ and $t=3$ are approximately 2 and 3 times the credibility for $t=1$, the variation of an individual insured's chances for an accident is not changing over time.

continued on next page

SOLUTION 2: Alternative interpretation, inconsistent with source paper

Assume the numbers given represent # of insureds with EXACTLY t years claims-free.

of claims at t = # of insureds claim-free at t × Expected Freq

Group	# of claims from			
	t=0	t=1	t=2	t=3
0.05	2,500	2,375	2,250	2,200
0.10	5,000	4,500	4,300	3,600
0.20	5,000	4,100	3,300	2,800
Total	12,500	10,975	9,850	8,600

t+ years Freq = Sum(# of claims from t or more years) / Sum(# of insureds for t or more years)

Total Freq = (12,500 + 10,975 + 9,850 + 8,600) / (125,000 + 113,000 + 104,500 + 94,000) = 0.0960

Credibility = 1 - Relative Freq to total

	Years claims-free			
	Total	1+ years	2+ years	3+ years
Freq	0.0960	0.0945	0.0929	0.0915
Relative Freq to total	1	0.9835	0.9677	0.9525
Credibility		0.0165	0.0323	0.0475
Relative Cred to total		1	1.955	2.875

Since the relative credibilities for 2+ and 3+ are fairly close to 2 and 3 times the credibility for 1+ years claims-free, the variation of an individual insured's chances for an accident is not changing over time.

19. 2017 Exam 8 - Q3 (1.5 points)

- (a) *I assume all premiums in this question are on-level. We want to use premium as the base instead of car-years if high frequency territories are also high average premium territories, and if territory rates are proper. In this part, we only need to look at the 2nd table given.*

There isn't enough information to know whether territories are priced correctly, but we can see that high frequency territories are not high premium territories, because territory C has the highest frequency but lowest average premium. As such, I would recommend using earned car-years as the base for frequency.

- (b) *The wording of this question says RELATIVE credibility, so based on the usage of this in the source material, I assume they are asking for the 3+ year credibility divided by the 1+ year credibility. That said, this should have been more explicit, and past exam question have been more explicit that we are asking for credibility relative to 1+ years claim-free.*

$$3 \text{ or more Freq} = (15 + 13.5) / (250 + 100) = 0.081$$

$$1 \text{ or more Freq} = (15 + 13.5 + 8) / (250 + 100 + 80) = 0.085$$

$$\text{Total Freq} = 47 / 500 = 0.094$$

$$3 \text{ or more Mod} = 0.081 / 0.094 = 0.866$$

$$1 \text{ or more Mod} = 0.085 / 0.094 = 0.903$$

$$3 \text{ or more Cred} = 1 - 0.866 = 0.134$$

$$1 \text{ or more Cred} = 1 - 0.903 = 0.097$$

$$\text{Relative Cred} = 0.134 / 0.097 = 1.379$$

20. 2018 Exam 8 - Q3 (2.75 points)

$$(a) R = \frac{1}{1-e^{-0.05}} = 20.504$$

$$\text{Mod} = (20.504)(0.038) + (1 - 0.038) = 1.741$$

The premium is given not at B rates, so we need to divide out current relativities to get premium at B rates.

Group	Prem at B Rates (\$000,000) = Prem/Curr Factor
A	360
X	180
Y	75
B	200
Total	815

$$1.741 = \frac{C/200}{(63000+C)/815}$$

$$(1.741) \frac{63000+C}{815} = \frac{C}{200}$$

$$0.4272(63,000 + C) = C$$

$$2.6918 + 0.4272C = C$$

$$C = 47,000$$

- (b) The wording here is a little odd, as there is no merit rating factor for the A+X category. Instead, there would be an experience mod that would replace the merit rating variable. So I think the question writer meant to ask what the indicated experience mod would be for a risk with 2 or more accident-free years. The answer to that is just the relative frequency for A+X to the total using premium at B rates.

$$\text{A+X Frequency} = (25 + 18) / (360 + 180) = 0.07963$$

$$\text{Total Frequency} = (63 + 47) / 815 = 0.13497$$

$$\text{Mod} = 0.07963 / 0.13497 = 0.59$$

- (c) Really, because of the wording of this question, there is only 1 main reason, which is that other variables aren't priced properly. The other item from the paper about high frequency territories having high premium is about exposure correlation existing in the first place, not about correcting for it. But I suppose if exposure correlation doesn't exist, then there is nothing to correct.

- If other rating variables are not priced properly, then EP would not properly account for exposure correlation with those variables.
- If there is no exposure correlation with other rating variables, then there will be no maldistribution issue to correct.

21. 2019 Exam 8 - Q3 (1.75 points)

(a) $Mod = ZR + (1 - Z)$

R is last year's relative claim frequency of current B ratings to the total class:

$$R = \frac{(\# \text{ of claims last year from current 'B' ratings}) / (\text{earned car years last year of current 'B' rating insureds})}{(\# \text{ of claims last year from class}) / (\text{earned car years last year of insureds in class})}$$

Since all claims in the class last year came from insureds that now have 'B' ratings, the numerators cancel out:

$$R = \frac{1 / (\text{earned car years last year of current 'B' rating insureds})}{1 / (\text{earned car years last year of insureds in class})} = \frac{\text{earned car years last year of insureds in class}}{\text{earned car years last year of current 'B' rating insureds}}$$

The above fraction is now just 1 divided by the portion of insureds that have 'B' ratings. Insureds with 'B' ratings had at least 1 claim last year, so we can get:

$$Pr(X \geq 1) = 1 - Pr(X = 0) = 1 - \left[\frac{(0+r-1)!}{0!(r-1)!} \times p^0 \times (1-p)^r \right] = 1 - [1 \times 1 \times (1-p)^r]$$

$$R = 1 / [1 - (1-p)^r] = 1 / [1 - (1-p)^{10}]$$

We can get p because we know the expected claim frequency will equal 0.101 and we have the negative binomial formula for the mean.

$$\frac{pr}{1-p} = \frac{10p}{1-p} = 0.101$$

$$\text{Solve for } p = 0.010$$

$$R = 1 / [1 - (1 - 0.010)^{10}] = 10.459$$

$$Mod = (0.02)(10.459) + (1 - 0.02) = 1.189$$

- (b) *The question wording asks you to describe why a CLASS may have less credibility, but the model solutions in the examiner's report discuss why an INDIVIDUAL RISK within a class might have less credibility. I'll show answers for both, even though credit was only given for the individual risk assumption (even though that's not what the question wording said).*

Solution 1: Talking about individual risk experience rating credibility

Individual risk experience rating credibility is used to distinguish between risks within a class. If the variance between risks in a class is low (i.e., risks within the class are very similar), then experience rating credibility will also be low regardless of the size of the class.

Solution 2: Talking about class credibility

Credibility depends not just on the volume of data, but on the variance of the data. So a class with lots of data could have more variance in loss results than a class with less data, and as such might deserve lower credibility.