An Actuarial Note on the Credibility of Experience of a Single Private Passenger Car & discussion of paper

Bailey & Simon, Hazam

LEARNING OBJECTIVES	KNOWLEDGE STATEMENTS
A1. Identify and evaluate possible rate classes. *Range of weight: 5-10 percent*	a. Characteristics of appropriate classifications b. Sampling techniques **c. Credibility Considerations** d. Statistical significance e. Cluster analysis
A2. Measure statistical significance of possible classes and estimate the loss costs of rating classes. *Range of weight: 5-10 percent*	a. Multidimensional relativities **b. Credibility techniques** c. Quintiles Test d. Holdout sample

By using data from the Canadian private passenger auto merit rating plan, Bailey & Simon demonstrate that a single car's experience for a single year has significant and measurable credibility for experience rating. They also show that the credibility of individual risk experience within a class varies based on how narrowly the class is defined. They aren't trying to set rates; they are looking backwards to show that the year Y-1 experience should be given some credibility in predicting the year Y experience.

Merit Rating Terminology

The merit rating system in Canada is similar to an experience rating plan, where the merit rating for an insured is based on the # of full years since the most recent accident (or if the insured has had no accidents, the # of years since the insured became licensed).

The merit ratings are:

1. 'A' - 3 or more years
2. 'X' - 2 years
3. 'Y' - 1 year
4. 'B' - 0 years

For example, if an insured had a merit rating of X at the start of year 1 but had 1 (or more) claims during year 1, then that insured would start year 2 as a B rating. If that insured instead had no claims in year 1, they would start year 2 with an A rating.

In the rating algorithm, this would be used as:

Premium = Base Rate × Merit Factor × Territory Factor × (any other variables)

The merit factors at the time of the paper were 0.65 for 'A', 0.80 for 'X', 0.90 for 'Y', and 1.00 for 'B'. This means that the base rate would correspond with 'B' risks.

In the paper, Bailey & Simon also use 'A + X' to mean 2 or more years and 'A + X + Y' as 1 or more years.

Experience Rating Formula

What Bailey & Simon do is ask: from a theoretical standpoint, what does the merit rating plan data imply about the credibility of experience from a single car for 1 year? They essentially do this by pretending that merit rating didn't exist, but instead, the rating plan had an experience modification factor. In this case, the premium formula would be:

Premium = Base Rate $\times$ Experience Mod $\times$ Territory Factor $\times$ (any other variables)

In experience rating, the experience mod is a credibility-weighted factor with credibility given to the experience of an individual risk and the complement of credibility given to the experience of the class of risks containing the individual risk. To put this in factor form, the experience is expressed relative to the class total experience. For example, using loss ratios as the measure of experience:

Cred-Wtd Individual Risk Loss Ratio = $Z \times$ Loss Ratio of Individual Risk + (1 - Z) $\times$ Loss Ratio of Class

Dividing all terms by the class loss ratio gives us the Mod formula:

Experience Mod = $Z \times \frac{\text{Loss Ratio of Risk}}{\text{Loss Ratio of Class}} + (1 - Z)$

Bailey & Simon denote the relative loss experience as R, so the formula becomes:

Mod = ZR + (1 - Z)

Normally in experience rating, we use historical data and established credibility values to calculate the Mod that will apply to a risk in the future policy term. Bailey & Simon will look at this backwards:

- They re-arrange the formula to solve for **Z** = (**Mod** - 1) / (**R** - 1).
- They start with the current policy term data to figure out what the ideal **Mod** would have been.
- Next they infer or estimate what **R** would have been for the prior policy term (they only use 1 year of experience since what they are trying to prove is that 1 year of data has some credibility, but in practice, experience rating often uses multiple years of historical data).
- Lastly, they plug **Mod** and **R** into the formula to solve for **Z**. The value of **Z** will then be the appropriate credibility for that individual risk for one year.

Bailey & Simon note that using relative loss ratios for **Mod** and **R** is usually problematic because severity across risks is generally too volatile and thus unreliable. While they do give an example of their calculation with relative loss ratios in Table 4 of their paper, the rest of their paper will omit severity and just use relative frequency TO PREMIUM (this would be equivalent to using relative loss ratios where severity is assumed to be constant across all risks, so it would cancel out).

Also, the premiums used in the relative frequency calculation should have 2 adjustments:

- Premiums should be on-leveled, so we don't double count the impact of past rate changes when calculating the experience Mod.
- Premiums should have the current merit rating factors backed out, since we will be replacing the merit rating factors with the experience Mod. Since 'B' ratings have a factor of 1.00, this is equivalent to saying premiums will be at 'B' rates.

Exposure Correlation

So why didn't Bailey & Simon calculate frequency as claim counts per exposure, the way we normally use the term? The reason is because of exposure correlation (what the paper calls "maldistribution"). Specifically, the authors were concerned about the possible exposure correlation between the territory rating variable and the merit rating variable.

As an example, if exposure correlation did exist between these variables, you might see more 'B' ratings in a given territory and more 'A' ratings in a different territory. This would mean that the frequency to car-years SHOULD differ between merit ratings solely because you'd expect some territories to have higher frequencies (to car-years) than other territories. However, when frequencies are defined relative to on-level premiums at 'B' rates, these premiums will include the current territory factors. If those territory factors are priced accurately, using premiums will adjust for the impact of exposure correlation.

To summarize the above using the paper's terminology, Hazam says a premium base (for frequency) only eliminates maldistribution if:

1. **High frequency (to car-year) territories are also high average premium territories.**
2. **Territorial (rate) differentials are proper.** One sign of this would be equal loss ratios across territories.

Calculating the Mod

We want to use data in the CURRENT period (2 policy years combined in the Bailey & Simon paper) to calculate the ideal **Mod** for a given risk. Instead of looking at individual risk data, Bailey & Simon calculate the ideal average Mod for all risks with a given current merit rating as:

$$\text{Mod} = \frac{\text{(\# of claims with rating) / (on-level earned premium for rating at 'B' rates)}}{\text{(\# of claims in total for class) / (class total on-level earned premium at 'B' rates)}}$$

For example, the Mod applied for 'B' ratings in class 1 on page 160 of the paper is 1.476. This is calculated based on numbers in Table 1 on page 162 as follows:

$$1.476 = \frac{\text{(\# of claims in class 1 from 'B' ratings) / (OLEP in class 1 from 'B' ratings at 'B' rates)}}{\text{(\# of claims in total for class '1') / (class '1' total OLEP at 'B' rates)}} = \frac{(37{,}730)/(17{,}226)}{(288{,}019)/(194{,}106)} = \frac{2.190}{1.484}$$

Bailey & Simon calculate Mods for 'A' and 'B' ratings, and instead of calculating Mods for 'X' and 'Y' ratings, Bailey & Simon calculate them for 'A + X' and 'A + X + Y'. This is because they want to show the impact of adding additional years claims-free.

This approach results in ideal Mod values for all ratings that, if used in the rating algorithm instead of merit rating factors, would produce equal ratios of frequency to earned premium across all merit ratings.

Note that if there were no other variables in the rating plan besides merit rating (i.e., no territory variable), there would be no issue with exposure correlation, and using premium or exposures as the denominators for the relative frequency calculation would give you identical results for the Mod values.

Calculating Mod Example

Suppose we have the following rates and data for a book of business:

Premium = 1,000 (base rate) $\times$ Territory Factor $\times$ Merit Rating Factor

Merit Rating	Factor
A	0.65
X	0.80
Y	0.90
B	1.00

Territory	Factor
1	1.00
2	0.75

Insured	Territory	Merit Rating at Start of Year 2	Year 2 Claim Count	Year 2 Prem	Year 2 Prem at B rates
1	1	A	0	650	1,000
2	1	B	1	1,000	1,000
3	1	X	0	800	1,000
4	1	B	2	1,000	1,000
5	2	B	1	750	750
6	2	B	0	750	750
7	2	X	1	600	750
8	2	Y	0	675	750

If pricing was "perfect", we would have equal ratios of claim counts to premium (not at B rates) across all merit ratings. We can determine the ideal mods (that will replace merit factors) needed to get this perfect premium by looking at frequency to premium that backs out the merit factors (i.e., at B rates). If desired, we can use these ideal mods and Premium = 1,000 (base rate) $\times$ Territory Factor $\times$ Mod to calculate what would have been "perfect" premiums for year 2, and we can confirm these are perfect by checking that the frequency to premium ratios are flat across all merit ratings:

(1) Year 2 Merit Rating	(2) Year 2 Prem at B rates	(3) Year 2 Claims	(4)=(3)/(2) Freq to Prem at B rates	(5)=(4)/(4tot) Relative Frequency	(6)=(2)×(5) Perfect Premium	(7)=(3)/(6) Frequency to Perfect Prem
A	1,000	0	0	0	0	n/a
X	1,750	1	0.0006	0.8	1,400	0.000714
Y	750	0	0	0	0	n/a
B	3,500	4	0.0011	1.6	5,600	0.000714
Total	7,000	5	0.0007	1	7,000	0.000714

For their purposes, Bailey & Simon instead present the data using A+X and A+X+Y, as they will use this to show the impact of adding additional years claim-free (Total is now A+X+Y + B):

(1) Year 2 Merit Rating	(2) Year 2 Prem at B rates	(3) Year 2 Claims	(4)=(3)/(2) Freq to Prem at B rates	(5)=(4)/(4tot) Relative Frequency aka Ideal **Mod**	(6)=(2)×(5) Perfect Premium	(7)=(3)/(6) Frequency to Perfect Prem
A	1,000	0	0	0	0	n/a
A+X	2,750	1	0.0004	0.509	1,400	0.000714
A+X+Y	3,500	1	0.0003	0.4	1,400	0.000714
B	3,500	4	0.0011	1.6	5,600	0.000714
Total	7,000	5	0.0007	1	7,000	0.000714

Calculating R

Next we need to obtain the values of **R**, which would be the ratios of relative frequency to premium from LAST YEAR'S experience for each individual risk. For any risks with CURRENT ratings of 'A', 'X', or 'Y', by definition they had no claims last year, so R = 0 for these risks (the same logic applies when defining risks as 'A + X' and 'A + X + Y'). For these risks, the Mod formula reduces to **Mod = 1 - Z**, and we can already solve for **Z = 1 - Mod**.

Any risk with a current 'B' rating would have had at least 1 claim last year (unless it was newly licensed). If we actually had data for last year's experience for current 'B' ratings, we would calculate last year's relative frequency to premium for current 'B' ratings in order to obtain **R**. However, if we don't have this data (and in the paper we don't), we can approximate **R** for 'B' ratings as follows:

- Assume that the class total claim frequency to earned car-years is the same each year.
- Assume that claim counts are Poisson distributed.

To see how the components of R are derived, remember that the mean of the Poisson process is λ, and the Poisson formula for the proportion of insureds with k accidents is:

$$Pr(X=k) = \frac{\lambda^k e^{-\lambda}}{k!} \qquad Pr(X=0) = \frac{\lambda^0 e^{-\lambda}}{0!} = e^{-\lambda} \qquad Pr(X \geq 1) = 1 - Pr(X=0) = 1 - e^{-\lambda}$$

Remember that all claims in the class last year only come from current 'B' ratings. To determine R for these insureds, Bailey & Simon use relative claim frequency to earned-car years, which we can estimate with the above assumptions. In this case, **R** is approximated as:

$$\text{R} = \frac{\text{(\# of claims last year from current 'B' ratings) / (earned car years last year of current 'B' ratings)}}{\text{(\# of claims last year for class) / (class total earned car years last year)}} = \frac{N\lambda/[N(1-e^{-\lambda})]}{N\lambda/N} = \frac{1}{1-e^{-\lambda}}$$

where λ = class total claim frequency = $\frac{\text{\# of claims from class in current year}}{\text{earned car years* of insureds in class in current year}}$

and N is the total earned car-years in the class last year.

*Note that for λ, car-years is used as the denominator instead of premium.

The **R** formula above can also be described as a ratio of the average number of claims produced by 'B' rating insureds to the average number of claims produced by any insured in the class. For example, the R applied for 'B' ratings in class '1' in page 160 of the paper is $\frac{1.044}{0.087} = 12$. This is calculated based on the R formula above with λ derived from Table 1 on page 162 as follows:

$$\lambda = \frac{\text{\# of claims from class '1'}}{\text{earned car years of insureds in class '1'}} = \frac{288{,}019}{3{,}325{,}714} = 0.087 \qquad \text{Avg \# claims from B ratings} = \frac{0.087}{1-e^{-0.087}} = 1.044$$

Finally, for 'B' ratings, we can re-arrange the Mod formula to solve for **Z** = (**Mod** - 1) / (**R** - 1). Continuing with the example from the prior page, we can get λ = 5 claims / 8 risks = 0.625, and then solve for Z:

Merit Rating	Mod	**R**	**Z** = (**Mod** - 1) / (**R** - 1)
A	0	0	1
A+X	0.509	0	0.491
A+X+Y	0.4	0	0.6
B	1.6	$\frac{1}{1-e^{-0.625}}$ = 2.152	0.521

What the Analysis Shows

Bailey & Simon show the implied credibilities for 'A' (3+ years), 'A + X' (2+ years), and 'A + X + Y' (1+ years) ratings for all classes in Table 2 of their paper. The credibility values for the 1 year ('A + X + Y' ratings) range between 0.038 and 0.071, which does imply that some credibility is warranted for the experience of a single car for a single year of claims-free experience (interestingly, you get negative credibility values if you just look at 'X' or 'Y' ratings by themselves, but this is because the Mod is relative to the class average, which is heavily influenced by 'A' ratings).

The 2nd to last column of Table 2 also shows the claim frequency for each class. All else being equal, higher frequency (more claim counts) would imply more data and thus greater credibility. However, when we look at the last column in Table 2 that shows the ratio of 3+ year credibility to claim frequency, we see that it is significantly higher for class 1. This is because experience rating credibility depends not just on the volume of data, but also the variance of loss experience within a class. Since classes 2 through 5 are more narrowly defined than class 1, risks within those classes will be more similar to each other than risks within class 1. As a result, experience rating, which distinguishes the individual risk from the class average risk, will have less credibility in classes 2-5.

Table 3 of the paper shows relative credibilities obtained by dividing the credibilities in Table 2 by the 1 year column in that table (e.g., dividing the credibility for A ratings by the credibility for A+X+Y). Bailey & Simon state that the closer the credibilities for 2 and 3 years of experience are to 2 and 3 times the 1 year credibility, then the less variation in an insured's probability of an accident for that class. They use an example to demonstrate this:

- Assume all risks have a mean claim frequency λ of either 0.05, 0.10, or 0.20.
- Assume frequency has a Poisson distribution, so $Pr(X = 0) = e^{-\lambda}$.

Claim		Expected number of risks with t years claims-free		
Frequency	$t = 0$	$t = 1$	$t = 2$	$t = 3$
0.05	100,000	95,123 = 100,000$\times e^{-0.05}$	90,484 = 95,123$\times e^{-0.05}$	86,071 = 90,484$\times e^{-0.05}$
0.10	100,000	90,484 = 100,000$\times e^{-0.10}$	81,873 = 90,484$\times e^{-0.10}$	74,082 = 81,873$\times e^{-0.10}$
0.20	50,000	40,937 = 50,000$\times e^{-0.20}$	33,516 = 40,937$\times e^{-0.20}$	27,441 = 33,516$\times e^{-0.20}$
Total	250,000	226,543	205,873	187,593

Claim	Expected number of claims in year following t from above risks			
Frequency	$t = 0$	$t = 1$	$t = 2$	$t = 3$
0.05	5,000 = 100,000$\times$0.05	4,756 = 95,123$\times$0.05	4,524	4,304
0.10	10,000 = 100,000$\times$0.10	9,048 = 90,484$\times$0.10	8,187	7,408
0.20	10,000 = 50,000$\times$0.20	8,187 = 40,937$\times$0.20	6,703	5,488
Total	25,000	21,992	19,415	17,200
Claim Frequency	0.10000 = 25,000 / 250,000	0.09708 = 21,992 / 226,543	0.09430	0.09169
Freq relative to $t = 0$	1	0.9708	0.9430	0.9169
Z = 1 - Rel Freq		0.0292	0.0570	0.0831
Relative Credibility		1	1.948	2.843

The relative credibilities of 1.948 and 2.843 are close to 2 and 3, respectively. However, Table 3 of the paper showed lower values, which the authors said could be due to risks entering/exiting the portfolio or risk characteristics changing over time. In the discussion of the paper, Hazam also points out that the credibility increases closely in proportion to the # of years only for low credibility values.

Recap of Conclusions

Bailey & Simon come to 3 conclusions:

1. The experience of a single car for 1 year has significant and measurable credibility for experience rating.
2. Individual risk experience is more credible when there is more variance in loss experience within a risk class, which occurs in less refined risk classification systems.
3. The credibilities for varying years of experience should increase in proportion to the # of years of experience, though this is only true if several conditions hold: credibilities are low, risks aren't entering/exiting the book, and risk characteristics aren't changing over time.

Note on Bühlmann Credibility

Hazam mentions the formula for Bühlmann credibility in his paper. Since it also appears on other papers on the syllabus, it is worth touching on here briefly.

Suppose X is a random variable (e.g., # of claims) with some distribution with parameter Θ (e.g., λ for a Poisson distribution), and Θ itself is a random variable with some distribution and additional parameters. In that case, the credibility of a sample of n observations from X is given by:

$$Z = \frac{n}{n+k}$$

n = # of claims in sample

$$k = \frac{E[Var(X|\Theta)]}{Var(E[X|\Theta])}$$

$E[Var(X|\Theta)]$ is known as the Expected Value of Process Variance (EPV)
$Var(E[X|\Theta])$ is known as the Variance of Hypothetical Means (VHM)

For example, if X is distributed Poisson(λ) and λ is distributed Normal(μ,σ^2), then:

$$k = \frac{E[Var(X|\lambda)]}{Var(E[X|\lambda])} = \frac{E[\lambda]}{Var(\lambda)} = \frac{\mu}{\sigma^2}$$

Note that k only needs to be calculated once for given random variables X and Θ, and remains constant for different samples of n sizes taken from the X variable. In the paper, Hazam backs into k by assuming $n = 100$ gives $Z = 0.046$, and then re-calculates Z for different values of n:

$Z_{100} = 0.046 = \frac{100}{100+k} \qquad \Longrightarrow \; k = 2{,}074$

$Z_{200} = \frac{200}{200+2{,}074} = 0.088 \qquad Z_{200}/Z_{100} = 0.088/0.046 = 1.912$

$Z_{300} = \frac{300}{300+2{,}074} = 0.126 \qquad Z_{300}/Z_{100} = 0.126/0.046 = 2.747$

This shows that even theoretically, the relative credibilities for 2 or 3 times the experience should be less than 2 and 3 times the original (single year) credibility.

Problem Knowledge Checklist

1. Experience Rating Formula
   - Be able to state whether you should use exposures or premium as the denominator of frequency when calculating the Mod, and why.
   - Be able to state the 2 conditions the paper lists for using premium to account for maldistribution (i.e., exposure correlation) between merit rating and territory.
   - Be able to calculate the Mod, R, and Z for each rating (including knowing the Poisson formulas for 'B' ratings).
   - Be able to derive and calculate R for B ratings for non-Poisson distributions.
2. What the Analysis Shows
   - Be able to briefly explain why individual risk experience is more credible when there is more variance in loss experience within a risk class.
   - Be able to use ratios of credibilities for 2 and 3 years claim-free to 1 year claim-free to compare the stability between books of business.
   - Be able to state why the ratios for 2 and 3 years would be less than 2 and 3, respectively.
3. Note on Bühlmann Credibility
   - Be able to calculate EPV and VHM to obtain k.
   - Be able to calculate the credibility using the Bühlmann credibility formula.

Past Exam Problems

1. **2000 Exam 9 - Q32 (3 points)**

   Based on Bailey and Simon's "An Actuarial Note on the Credibility of Experience of a Single Private Passenger Car" and the table below, answer the following.

Private Passenger Automobile Liability - Non-Farmers Class 3 - Business Use					
Merit Rating	Earned Car Years	Earned Premium at Present B Rates	Number of Claims Incurred	Claim Frequency per $1,000 of Premium	Relative Claim Frequency
A	247,424	$25,846,000	31,964	1.237	0.920
X	15,868	$1,783,000	2,695	1.511	1.123
Y	20,369	$2,281,000	3,546	1.555	1.156
B	37,666	$4,129,000	7,565	1.832	1.362
Total	321,327	$34,039,000	45,770	1.345	1.000

   where: Class A - Three or more years claim free
   Class X - Two years claim free
   Class Y - One year claim free
   Class B - Zero years claim free

   (a) (1.5 points)

   Calculate the credibilities for a single private passenger car for one or more years, two or more years, and three or more years claim-free. Show all work.

   (b) (0.5 point)

   Briefly describe the relationship that Bailey and Simon expect between the three credibilities from part (a).

   (c) (1 point)

   Do the credibilities in part (a) follow the relationship described in part (b)? Briefly explain why or why not.

2. **2001 Exam 9 - Q2 revised (1 point)**

   According to Bailey and Simon's "An Actuarial Note on the Credibility of Experience of a Single Private Passenger Car," which of the following is false?

   (a) The experience for one car for one year has significant and measurable credibility for experience rating.

   (b) Credibility for experience rating depends on the variation of individual hazards within the class.

   (c) In a highly refined private passenger rating classification system that reflects inherent hazard, there would not be much accuracy in an individual risk merit rating plan.

   (d) In experience rating, an increase in the volume of data in the experience period increases the reliability of the indication in proportion to the square root of the volume.

3. **2001 Exam 9 - Q22 (2.5 points)**

   Use Bailey and Simon's "An Actuarial Note on the Credibility of Experience of a Single Private Passenger Car," and Hazam's discussion to answer the following questions.

   (a) (1.5 points)

   Using the information below, calculate the credibility for 1-year and 2-year claim-free periods for Class 1. Show all work.

	Number of Years Claim Free	Earned Premium at Present Rates	Number of Claims Incurred	Earned Car Years
Class 1	2 or more	$5,000,000	7,000	15,000
	1	$7,000,000	10,000	12,250
	0	$1,000,000	2,000	400
	Total	$13,000,000	19,000	27,650

   (b) (0.5 point)

   What exposure base do the authors use? Explain why.

   (c) (0.5 point)

   According to Hazam, what two conditions must be met to use the exposure base described in part (b)?

4. ***Good problem*** **2002 Exam 9 - Q47 (2 points)**

   (a) (1.5 points)

   Given the following data, calculate the credibilities for 1-year and 2-year claim free periods.

   A represents 3 or more years since the most recent accident.
   X represents 2 years since the most recent accident.
   Y represents 1 year since the most recent accident.
   B represents 0 years since the most recent accident.

	Earned Car Years	Earned Premium at Present Class B Rates	Number of Claims
A	50,000	$5,500,000	5,000
X	6,500	$682,500	1,000
Y	5,000	$535,000	850
B	4,500	$490,500	900
TOTAL	66,000	$7,208,000	7,750

   (b) (0.5 point)

   Give two possible reasons that the 2-year credibility is less than 2 times the 1-year credibility.

5. **2003 Exam 9 - Q2 (1 point)**

   Which of the following statements is <u>false</u> for private passenger auto experience rating?

   (a) Credibility assigned to an individual risk within a highly refined classification rating plan would be higher than the credibility assigned in a less refined rating plan.
   (b) Credibility for experience rating depends on the amount of variation individual hazard within the class.
   (c) Credibility for experience rating is significant and measurable when based on data from one car for one year.
   (d) Credibility within a highly refined private passenger classification rating system would be larger where a wide range of hazard is encompassed within a classification.

6. **2003 Exam 9 - Q22 (3 points)**

   You are given the following data:

Class	Years since last accident	Actual Earned Premium at Present B Rates	Earned Car Years	Number of Claims
A	3+	375,000	2,500	200
X	2	15,000	100	12
Y	1	22,500	150	20
B	0	37,500	250	38

   Assume that the same rate is charged to all insureds within a class, and there have been no rate changes in or since the experience period.

   (a) (1 point)

   What is the credibility of 3 or more accident-free years of experience?

   (b) (1 point)

   What is the credibility of 1 or more accident-free years of experience?

   (c) (1 point)

   Give two possible reasons why the answer in part (a) is not 3 times the answer in part (b).

7. **2004 Exam 9 - Q2 revised (1 point)**

   Given the following information:

Class	Number of Years Since Most Recent Accident	Earned Car Years	Earned Premium at Present B Rates	Number of Claims
A	3 or more	10,000	$1,000,000	1,000
X	2	7,000	$770,000	1,155
Y	1	5,000	$625,000	1,250
B	0	2,000	$400,000	1,000
Total		24,000	$2,795,000	4,405

   Calculate the credibility of one or more accident-free years of experience.

8. *Good problem* **2005 Exam 9 - Q3 (3 points)**

   (a) (2 points)

   Given the following information:

   N = the number of drivers in the population
   m = the mean claim frequency of all drivers
   Mod = the credibility weighted modification factors for risks with one or more claims in the past year

   Derive the formula for the credibility assigned to the experience of drivers with one or more claims in the past year.

   Assume that claim frequency follows a Poisson distribution.

   (b) (1 point)

   If there is a switch from a less refined class plan to a highly refined class plan, describe the likely change in the credibility assigned to an individual risk.

9. **2006 Exam 9 - Q2 (4 points)**

   (a) (3 points)

   Given the following information about an automobile insurance portfolio:

Group	Number of Accident-Free Years	Earned Premium at Present Group D Rates	Number of Claims Incurred
A	3	$25,000,000	40,000
B	2	$8,000,000	15,000
C	1	$13,000,000	25,000
D	0	$8,000,000	30,000

   Calculate the credibility of a single car for each of the following: one-year, two-year, and three-year accident-free periods.

   (b) (1 point)

   In performing the analysis in part (a) above, would using car years instead of earned premium as an exposure base be more preferable? Explain why or why not.

10. ***Good problem*** **2007 Exam 9 - Q2 (3.5 points)**

(a) (2 points)

The following data were compiled from the ABC automobile insurance portfolio:

Group	Number of Accident-Free Years	Earned Premium at Present Group D Rates	Number of Claims Incurred
A	3 or more	$100,000,000	120,000
B	2	$10,000,000	25,000
C	1	$17,000,000	44,000
D	0	$10,000,000	36,000

Calculate the credibility of a single car for each of the following ranges of accident-free years:

i 1 or more
ii 2 or more
iii 3 or more

(b) (1 point)

The following table provides the single car credibility for the XYZ automobile insurance portfolio:

Accident-Free Years	Single Car Credibility
1 or More	0.14
2 or More	0.10
3 or More	0.06

Discuss two conclusions that can be drawn from the different credibility results of the ABC and XYZ portfolios.

(c) (0.5 point)

Explain why analysis of the two portfolios with different classification plans could assign different values to the credibility of the experience of a single car.

11. ***Good problem*** **2008 Exam 9 - Q5 (2 points)**

A liability insurer collects the following data for a particular class of private passenger auto risks:

Accident-Free Years	Earned Exposures	Incurred Losses($)
2 or more	2,500	1,000,000
1	500	500,000
0	1,000	2,500,000
Total	4,000	4,000,000

Assume the following:

- The base rate is $1,250 per exposure.
- An experience rating factor is the only factor applied to the base rate.

(a) (1 point)

Calculate the credibility of an exposure that is accident-free for 1 or more years.

(b) (1 point)

Calculate the premium for an exposure that is accident-free for 2 or more years.

12. ***Good problem*** **2009 Exam 9 - Q4 (3.5 points)**

The following information can be used to calculate the credibility assigned to the experience of a single private passenger car.

Group	Last Accident	Earned Car Years	Premium at Present B Rates	Number of Claims
A	3 or more	650,000	400,000,000	50,000
X	2	230,000	150,000,000	20,000
Y	1	100,000	75,000,000	12,000
B	0	M	45,000,000	18,000
Total		980,000 + M	670,000,000	100,000

Assume claim counts follow a Poisson distribution.

(a) (2.5 points)

Calculate M, the earned car years for Group B, given that the credibility for an insured that has had no claim-free years is equal to 0.167.

(b) (1 point)

Calculate the credibility for the group of risks that have been claim-free for two or more years.

13. **2010 Exam 9 - Q5 (1 point)**

An insurance company has a private passenger auto book of business with the following claims experience:

Group	Number of Accident-Free Years	Earned Premium at Present Group D Rates	Number of Claims Incurred
A	3 or more	60,000,000	45,000
B	2	15,000,000	15,000
C	1	20,000,000	29,300
D	0	5,000,000	18,700
		100,000,000	108,000

Calculate the credibility of a single car for a driver with one or more accident-free years.

14. ***Good problem*** **2011 Exam 8 - Q1 (3 points)**

An insurance company is using a merit rating plan for drivers in two states. State X has the following claims experience:

Group	Number of Accident-Free Years	Earned Premium at Present Group D rates	Number of Claims Incurred
A	3 or more	$500,000	240
B	2	$150,000	125
C	1	$200,000	190
D	None	$300,000	300
Total		$1,150,000	855

State Y has the following relative claim frequencies for accident-free experience:

Number of Accident-Free Years	Relative Claim Frequencies to Total
3 or more	0.70
2 or more	0.77
1 or more	0.84

Assuming that no new risks enter or leave either state, use relative credibility to explain which state has more variation in an individual insured's probability of an accident.

15. ***Good problem* 2012 Exam 8 - Q6 (2.5 points)**

An insurance company has a private passenger auto book of business with the following claims experience:

Territory	Years Since Last Accident	Earned Premium at Present Rates for Two Years Since Last Accident	Earned Car Years	Number of Claims	Incurred Loss
1	0	$15,000,000	15,000	5,000	$9,000,000
1	1	$125,000,000	125,000	41,000	$75,000,000
1	2+	$230,000,000	230,000	76,000	$138,000,000
2	0	$25,000,000	25,000	7,000	$16,000,000
2	1	$310,000,000	300,000	84,000	$187,000,000
2	2+	$550,000,000	535,000	147,000	$328,000,000
3	0	$10,000,000	10,000	4,000	$7,000,000
3	1	$80,000,000	100,000	35,000	$43,000,000
3	2+	$160,000,000	170,000	60,000	$100,000,000

Choose an appropriate exposure base for calculating credibility. Justify the selection.

16. ***Good problem* 2014 Exam 8 - Q5 (2.5 points)**

The following data shows the experience of a merit rating plan for a specific state.

Number of Accident-Free Years	Earned Car Years	Earned Premium ($000)	Number of Incurred Claims
3 or More	250,000	250,000	1,200
2	300,000	100,000	625
1	25,000	100,000	750
0	12,000	150,000	1,500
Total	587,000	600,000	4,075

The base rate is $1,000 per exposure. No other rating variables are applicable.

(a) (0.5 point)

The typical exposure base used to develop the merit rating plan is earned premium. Briefly discuss two assumptions in selecting this exposure base.

(b) (1.5 points)

Calculate the ratio of credibility for an exposure with two or more years accident-free experience to one or more years accident-free experience.

(c) (0.5 point)

Calculate the premium for an exposure that is accident free for two or more years.

17. *Good problem* **2015 Exam 8 - Q1 (2.5 points)**

An actuary is evaluating a merit rating plan for private passenger cars. Given the following:

Number of Accident-Free Years	Earned Car Years	Number of Claims Incurred
2 or More	500,000	20,000
1	200,000	15,000
0	100,000	9,000
Total	800,000	44,000

- Frequency varies by territory.
- State law prohibits reflecting territory differences in rating.
- Annual claims for an individual driver follow a Poisson distribution.
- Claim cost distributions are similar across all drivers.

(a) (0.5 point)

Identify one potential issue with the exposure base used. Briefly explain whether or not earned premium would be a better choice for the exposure base.

(b) (1 point)

Calculate the credibility of one driver with one or more year's accident-free experience.

(c) (1 point)

Calculate the credibility of one driver with 0 Accident-Free years.

18. **2016 Exam 8 - Q1 (2.75 points)**

A group of insureds have different expected claim frequencies. The number of insureds claim-free for the past t years is as follows:

Expected Claim Frequency	**t=0**	**t=1**	**t=2**	**t=3**
0.05	50,000	47,500	45,000	44,000
0.10	50,000	45,000	43,000	36,000
0.20	25,000	20,500	16,500	14,000
Total	125,000	113,000	104,500	94,000

Determine whether the variation of an individual insured's chance for an accident changes over time.

19. ***Good problem*** **2017 Exam 8 - Q3 (1.5 points)**

The following data shows the experience of a merit rating plan for private passenger vehicles. The merit rating plan uses multiple rating variables, including territory.

Number of Accident-Free Years	Earned Car Years (000s)	Earned Premium ($000s)	Number of Incurred Claims
5 or More	250	500,000	15,000
3 and 4	100	90,000	13,500
1 and 2	80	60,000	8,000
0	70	50,000	10,500
Total	500	700,000	47,000

Territory	Frequency	Average Premium
A	0.05	1,500
B	0.10	2,000
C	0.15	1,250

(a) (0.75 point)

Recommend and justify an exposure base for this merit rating plan.

(b) (0.75 point)

Calculate the relative credibility of an exposure that has been three or more years accident-free using the exposure base from part (a) above.

20. **2018 Exam 8 - Q3 (2.75 points)**

An insurance company has a private passenger auto book of business with the following claims experience:

Group	Number of Accident-Free Years	Earned Premiums	Current Merit Rating Factor	Number of Claims Incurred
A	3 or more	216,000,000	0.60	25,000
X	2	135,000,000	0.75	18,000
Y	1	63,750,000	0.85	20,000
B	0	200,000,000	1.00	C
Total		614,750,000		63,000 + C

- Claim counts follow a Poisson distribution with parameter $\lambda = 0.05$.
- The credibility for the new policy period for an insured that has had no claim-free years is equal to 0.038.

(a) (1.5 points)

Calculate C, the number of claims incurred for Group B.

(b) (0.75 point)

Calculate the merit rating factor for an exposure that is accident-free for two or more years for the new policy period.

(c) (0.5 point)

Briefly explain two circumstances under which using earned premium as the exposure base would not correct for maldistribution.

21. *Good problem* **2019 Exam 8 - Q3 revised (1.75 points)**

    An insurance company has a private passenger auto book of business with an experience modification factor in its rating plan.

    Given the following:

    - Annual claims for an individual driver follow a negative binomial distribution with r = 10.
    - The expected claim frequency for the entire book of business is 0.101.
    - The credibility for the group of risks that have had at least one accident in the last year is 0.02.

    For the negative binomial distribution:

    - $f(x) = \binom{x+r-1}{x}(1-p)^r p^x$
    - $E[X] = \frac{pr}{1-p}$

    (a) (1.25 points)

    Calculate the experience modification factor for a policy that has had at least one accident in the last year.

    (b) (0.5 point)

    Describe why a class with a higher volume of claims and more exposures may have less credibility than a class with fewer claims and exposures.

Solutions to Past Exam Problems

1. **2000 Exam 9 - Q32 (3 points)**

   (a)

Rating	Premium ($000)	Claim Counts	Counts/Prem	Freq Relative to Total	Credibility
A	25,846	31,964	1.237	0.920	0.080
A+X	27,629	34,659	1.254	0.933	0.067
A+X+Y	29,910	38,205	1.277	0.950	0.050
Total	34,039	45,770	1.345	1	

   (b) The credibilities should increase in proportion to the # of years of experience if the chance of accidents for individual risks remains constant and no risks enter or leave the class.

   (c) 2 Years relative to 1 year: 0.067 / 0.050 = 1.338
   3 Years relative to 1 year: 0.080 / 0.050 = 1.590

   Since these are much less than 2 and 3, respectively, it must be that risks' chances for accidents are changing and/or risks may be entering or leaving the class.

2. **2001 Exam 9 - Q2 revised (1 point)**

   (a) True: this is one of the main points of the Bailey & Simon paper.
   (b) True: the more difference between individual risks in a class, the more powerful individual risk rating will become.
   (c) True: if the variance in loss experience between risks is largely explained by the classification rating variables, then experience rating wouldn't add much predictive power.
   (d) FALSE: This is in contrast to Bailey & Simon's 3rd conclusion.

3. **2001 Exam 9 - Q22 (2.5 points)**

   (a) I assume the problem meant 1 or more years and 2 or more years claim-free, since we are not given data for exactly 2 years.

   I also assume the premium is at present 0 years claim-free rates.

   Note that table of information all reflects current year data, which we will use to determine the Mod. We don't have prior year data, but we know R will equal 0 for both 1+ and 2+ years claim-free.

   1+ years Mod = $\frac{(7{,}000+10{,}000)/(\$5{,}000{,}000+\$7{,}000{,}000)}{19{,}000/\$13{,}000{,}000}$ = 0.969

   Since R = 0 for 1+ years claim-free, Z = 1 - Mod = 1 - 0.969 = 0.031

   2+ years Mod = $\frac{7{,}000/\$5{,}000{,}000}{19{,}000/\$13{,}000{,}000}$ = 0.958

   Since R = 0 for 2+ years claim-free, Z = 1 - Mod = 1 - 0.958 = 0.042

   (b) The authors use earned premium at current class B rates as their exposure base to avoid the maldistribution caused when higher frequency territories produce more X, Y, and B risks and higher premiums.

   (c)
   - High frequency territories are also high average premium territories.
   - Territorial rate differentials are proper.

4. **2002 Exam 9 - Q47 (2 points)**

   Note: the terminology here for "class" is different than in the source paper. In the source paper, all of the merit ratings have different factors, but can be contained within a single broader class.

   (a) **Solution assuming 1+ and 2+ years claim-free (more in line with the paper and what the CAS intended):**

   I assume the problem meant 1+ and 2+ years claim-free, and not exactly 1 and 2 years claim-free.

   A+X+Y Mod = $\frac{(850+1{,}000+5{,}000)/(\$535{,}000+\$682{,}500+\$5{,}500{,}000)}{7{,}750/\$7{,}208{,}000} = 0.948$

   A+X+Y Credibility = 1 - 0.948 = 0.052

   A+X Mod = $\frac{(1{,}000+5{,}000)/(\$682{,}500+\$5{,}500{,}000)}{7{,}750/\$7{,}208{,}000} = 0.903$

   A+X Credibility = 1 - 0.903 = 0.097

   Solution for EXACTLY 1 and 2 years claim-free:

   Y Mod = $\frac{850/\$535{,}000}{7{,}750/\$7{,}208{,}000} = 1.478$

   Y Credibility = 1 - 1.478 = -0.478

   X Mod = $\frac{1{,}000/\$682{,}500}{7{,}750/\$7{,}208{,}000} = 1.363$

   X Credibility = 1 - 1.363 = -0.363

   Note that the credibilities are negative. The reason for this is that the Mod is applied to the class rate, and the class mostly consists of the 3+ years claims-free insureds. As such, if the class rate is set to be the weighted average loss cost for all insureds in the class, the class rate will be low, and the factors for X and Y ratings will be higher than the class rate. So for X and Y insureds, the Mod $>$ 1, and R = 0 since they were claim-free last year, thus a negative credibility results.

   (b)
   - Risks are entering/exiting the portfolio.
   - Risk characteristics are changing over time.

5. **2003 Exam 9 - Q2 (1 point)**

   (a) FALSE: the more refined the classification plan, the less need for experience rating. In other words, if all the variation in losses is explained through rating variables, then using an individual risks's past experience adds no value.

   (b) True: The more variation within the class, the more meaningful experience rating will be, since classification rating does not sufficiently explain the variation in losses between risks.

   (c) True: This is a conclusion of the Bailey & Simon paper.

   (d) True: This is basically the same as (b) above.

6. **2003 Exam 9 - Q22 (3 points)**

   Note: the terminology here for "class" is different than in the source paper. In the source paper, all of the merit ratings have different factors, but can be contained within a single broader class.

   (a) Average Frequency = $\frac{200+12+20+38}{375{,}000+15{,}000+22{,}500+37{,}500} = 0.0006$

   Class A Frequency $= 200/375,000 = 0.000533$

   Mod = $0.000533/0.0006 = 0.8889$

   $Z_A = 1 - 0.8889 = 0.1111$

   (b) Class A + X + Y Frequency = $\frac{200+12+20}{375{,}000+15{,}000+22{,}500} = 0.00056$

   Mod = $0.00056/0.0006 = 0.9374$

   $Z_{A+X+Y} = 1 - 0.9374 = 0.0626$

   (c)
   - Risks are entering/exiting the portfolio.
   - Risk characteristics are changing over time.

7. **2004 Exam 9 - Q2 revised (1 point)**

   Note: the terminology here for "class" is different than in the source paper. In the source paper, all of the merit ratings have different factors, but can be contained within a single broader class.

   Mod = $\frac{(1{,}250+1{,}155+1{,}000)/(\$625{,}000+\$770{,}000+\$1{,}000{,}000)}{4{,}405/\$2{,}795{,}000}$ = 0.902

   Z = 1 - 0.902 = 0.098

8. **2005 Exam 9 - Q3 (3 points)**

   (a) Let X = # of claims

   Pr (risk was accident free last year) = $Pr(X = 0) = e^{-m}$
   Pr (risk had at least one accident last year) = $1 - Pr(X = 0) = 1 - e^{-m}$

   # of risks accident free last year = $N \times Pr(X = 0) = N(e^{-m})$
   # of risks with at least one accident last year; $N \times Pr(X > 0) = N(1 - e^{-m})$

   expected # of claims last year = Nm
   freq = avg # of claims last year for current B risks = $(Nm)/[N(1 - e^{-m})] = m/(1 - e^{-m})$

   R = (freq for B)/(overall frequency) = $(m/(1 - e^{-m}))/m = (1/(1 - e^{-m}))$

   $Z = (M - 1)/(R - 1) = (M - 1)/((1/(1 - e^{-m})) - 1)$

   (b) Credibility for an individual risk is lowered when the class plan is highly refined, because it is more difficult to identify differences in the loss potential for the particular risk at-hand from the average risk in the class.

9. **2006 Exam 9 - Q2 (4 points)**

   (a) **Solution assuming 1+, 2+, and 3+ years claim-free (more in line with the paper):**

   I assume the problem meant 1+, 2+, and 3+ years claim-free, and not exactly 1, 2, and 3 years claim-free.

Group	EP(000)	Claims	Freq	Mod = Freq / Total Freq	Cred = 1-Mod
3+	25,000	40,000	40/25 = 1.6	0.785	21.5%
2+	33,000	55,000	55/33 = 1.667	0.818	18.2%
1+	46,000	80,000	80/46 = 1.739	0.854	14.6%
Total	54,000	110,000	110/54 = 2.037		

   Solution for EXACTLY 1, 2, and 3 years claim-free:

   Total Frequency = $\frac{40{,}000+15{,}000+25{,}000+30{,}000}{\$25{,}000{,}000+\$8{,}000{,}000+\$13{,}000{,}000+\$8{,}000{,}000}$ = 0.00204

   1 Year Mod = $\frac{25{,}000/\$13{,}000{,}000}{0.00204}$ = 0.944

   1 Year Credibility = 1 - 0.944 = 0.056

   2 Year Mod = $\frac{15{,}000/\$8{,}000{,}000}{0.00204}$ = 0.920

   2 Year Credibility = 1 - 0.920 = 0.080

   3 Year Mod = $\frac{40{,}000/\$25{,}000{,}000}{0.00204}$ = 0.785

   3 Year Credibility = 1 - 0.785 = 0.215

   (b) Using premium is preferable as it will account for any exposure correlation with other variables like territory.

10. **2007 Exam 9 - Q2 (3.5 points)**

    (a) 1+ years frequency = (120 + 25 + 44) / (100 + 10 + 17) = .149%
    2+ years frequency = (120 + 25) / (100 + 10) = .132%
    3+ years frequency = 120 / 100 = .12%
    Overall frequency = 225,000 / 137M = .164%

    i 1 or more Z = 1 - .149/.164 = 9.1%
    ii 2 or more Z = 1 - .132/.164 = 19.5%
    iii 3 or more Z = 1 - .12/.164 = 26.8%

    (b) *The declining credibility for increased years of experience for XYZ is unusual, but not theoretically impossible. The focus here is on relative credibilities and magnitude of the credibilities.*

    Relative credibilities for ABC are nearly 1:2:3, but very different for XYZ. Insurer XYZ may have more risk entering and leaving classes than ABC.

    You could have instead suggested risks characteristics are changing over time for XYZ.

    Insurer XYZ's class plan may be more refined since the resulting credibilities are lower than ABC's (assuming both portfolios have equal total frequency).

    A higher overall frequency would imply higher credibilities, so this could also be a reason for the difference between books.

    (c) If one portfolio has a more refined class plan then the credibility assigned to the experience of a single car would be lower relative to the other portfolio which has a less refined plan (assuming both portfolios have equal total frequency).

11. **2008 Exam 9 - Q5 (2 points)**

    (a) *Note: in this problem, we cannot use relative frequency as we would for other problems to obtain the Mod since we don't have claim counts. As such, we have to use relative pure premium.*

    $\text{Mod} = \frac{(\$500{,}000+\$1{,}000{,}000)/(500+2{,}500)}{\$4{,}000{,}000/4{,}000} = 0.5$

    Z = 1 - 0.5 = 0.5

    (b) $\text{Mod} = \frac{(\$1{,}000{,}000)/2{,}500}{\$4{,}000{,}000/4{,}000} = 0.4$

    Premium = Base Rate $\times$ Mod = (\$1,250)(0.40) = \$500

12. **2009 Exam 9 - Q4 (3.5 points)**

    (a) $Mod = ZR + (1 - Z)$

    Mod = (18,000 / 45,000) / (100,000 / 670,000) = 2.68

    2.68 = 0.167R + (1 - 0.167) $\implies$ R = 11.05988

    $11.05988 = 1/(1 - e^{-\lambda})$ $\implies$ λ = 0.09477

    0.09477 = 100,000 / (980,000 + M) $\implies$ M= 75,198.40

    (b) Mod = $\frac{(50+20)/(400+150)}{100/670}$ = 0.85273

    Z = 1 - Mod = 0.14727

13. **2010 Exam 9 - Q5 (1 point)**

    $Mod = \frac{(29{,}300+15{,}000+45{,}000)/(20M+15M+60M)}{108{,}000/100M} = 0.87$

    $Z = 1 - Mod = 0.13$

14. **2011 Exam 8 - Q1 (3 points)**

This question is really asking about which state has more variation in accident probabilities OVER TIME (i.e., not the variation within each book).

State X # of yrs clm free	EP	# clms	Rel. Clm Free (M)	Z = 1 - M	n yr Z / 1 yr Z
3+	500,000	240	$\frac{240/500{,}000}{855/1{,}150{,}000} = 0.6456$	0.354	2.90
2+	650,000	365	$\frac{365/650{,}000}{855/1{,}150{,}000} = 0.755$	0.245	2.00
1+	850,000	555	$\frac{555/850{,}000}{855/1{,}150{,}000} = 0.878$	0.122	1.00
0	300,000	300			
	1,150,000	855			

State Y	Mod	Z = 1 - M	n yr Z / 1 yr Z
3+	0.70	0.30	1.875
2+	0.77	0.23	1.438
1+	0.84	0.16	1.00

State X's n yr Z / 1 yr Z ratio is closer to 3, 2, 1 for 3+, 2+, 1+

- State X is more stable over time
- State Y has more variation over time

15. **2012 Exam 8 - Q6 (2.5 points)**

The graders were looking for a solution here based on the text, so they wanted you to check how frequency correlates with premiums and whether loss ratios were flat. Based on that, they wanted you to conclude that you should use car-years as the base. That said, so long as territory relativities are proper, it would be fine to use premium as the base, though the graders didn't give credit for that.

Premium should be used as the exposure base to prevent the maldistribution of premium if higher frequency territories have higher premiums and territory relativities are proper. Testing this with the data shows:

Territory	Frequency ($\sum claims / \sum$ car years)	avg Prem ($\sum Prem / \sum$ car years)	Loss Ratio
1	0.330	1,000	0.6
2	0.277	1,029	0.6
3	0.354	893	0.6

All territories have the same Loss Ratio, which suggests the territory relativities are proper. However, higher frequency territories do not have higher average premiums. Therefore, it is advisable to use earned car years as the exposure base instead of earned premium.

16. **2014 Exam 8 - Q5 (2.5 points)**

There is an issue with the solution in the examiner's report for this problem. The question itself isn't inherently flawed, so it isn't a defective question (that said, it is a bit unusual to see a higher average premium and higher frequency for people with 3+ years accident free compared to only 2 years accident free).

Using earned premium as the exposure base is basically akin to using the loss ratio method versus the pure premium method to price relativities. When there is only 1 rating variable in the rating algorithm, there will be no exposure correlation between rating variables, and both approaches would give identical results. In other words, in this problem, since the merit plan is the only rating variable, using exposures or premium should give you the exact same answer if done correctly. So the graders should have given full credit to using exposures in part (b) and using the Mod calculated based on exposures in part (c).

You could have also solved this problem using premium, but not in the way shown in the examiners report, and it would have taken longer than using exposures. If you were to use premium, the premium needs to be at a common level (I assume it is already at the current rate level). You can do this by first calculating the current relativities by dividing the premiums by the base rate of $1,000 and then by the exposures. You'll get relativities of 1, 0.33, 4, and 12.50 for 3+, 2, 1, and 0 years, respectively. Then you can divide the premium numbers by these relativities to obtain premiums at the common level of 3+ years claims-free. Then you can use these premiums at the common level to calculate frequencies, and you'll end up with the exact same answers to parts (b) and (c) as if you had used exposures as the base instead of premiums.

(a) i. High frequency territories are also high average premium territories.
ii. Territorial differentials are proper.

(b) 2+ years frequency (to ECY) = (1,200 + 625) / (250,000 + 300,000) = 0.0033
1+ years frequency (to ECY) = (1,200 + 625 + 750) / (250,000 + 300,000 + 25,000) = 0.0045
Total frequency (to ECY) = 4,075 / 587,000 = 0.0069

2+ years Mod = 0.0033 / 0.0069 = 0.4780
1+ years Mod = 0.0045 / 0.0069 = 0.6451

2+ years Credibility = 1 - 0.4780 = 0.5220
1+ years Credibility = 1 - 0.6451 = 0.3550

Ratio of 2+ to 1+ Credibility = 0.5220 / 0.3550 = 1.471

(c) Premium = $1,000 $\times$ 0.4780 = $478.00

17. **2015 Exam 8 - Q1 (2.5 points)**

    (a) We would want to use earned premium as an exposure base if there is exposure correlation between territory and number of years accident-free and territory relativities are properly priced. There may be correlation since frequency varies by territory, but since territories are not properly priced due to not being allowed in rating, premium will not be an improvement over earned car years as an exposure base.

    To clarify, assuming merit rating is the only rating variable (since we are told territory is not used for rating and we aren't told about any other rating variables), using either exposures or premium would work equally well in this case since there would be no exposure correlation with other rating variables (as there are none with this assumption). However, if there are additional variables in the rating plan that do have exposure correlation with merit rating, then using premium would definitely be an improvement over using exposures.

    (b) *1+ yrs R = 0 by definition since 1+ years accident free. That means Mod = Relative Frequency = 1 - Z.*

    1+ yrs Freq = 35 / 700 = 0.05

    Total Freq = 44 / 800 = 0.055

    1+ yrs Mod = 0.05 / 0.055 = 0.9091

    1+ yrs Cred = 1 - 0.9091 = 0.0909

    (c) 0 yrs R = 1 / (1 - $e^{-0.055}$) = 18.686

    0 yrs Freq = 9 / 100 = 0.09

    0 yrs Mod = 0.09 / 0.055 = 1.6364

    0 yrs Cred = (1.6364 - 1) / (18.686 - 1) = 0.0360

18. **2016 Exam 8 - Q1 (2.75 points)**

This table is clearly related to the table in Appendix 1 of the Bailey & Simon paper. However, the purpose of that table is to demonstrate that with a fixed cohort of risks with constant frequencies, you can give about twice as much individual risk credibility to a risk with 2+ years-claims free compared to a risk with 1+ years claims-free (and about 3 times the credibility for 3+ years). In other words, the lack of variation of an individual insured's chance of an accident is taken as a given in creating that table, rather than learned as a conclusion from the table. So assuming the group of insureds in this question has no one enter or leave the group, then the makeup of the group is constant, and with constant expected frequencies, an individual insured randomly chosen from the group would have the same expected frequency no matter when the insured is chosen in time (note that insureds with claims don't actually leave the group, so they would still be included in calculating the expected frequency for the group). Thus, of course the variation of an individual insured's chance for an accident won't change over time - as mentioned above, this is already assumed when you have a fixed cohort of risks with constant frequencies. It seems like the question writer didn't understand this point, and as such, I believe you'll have to essentially work backwards by seeing whether the 2+ and 3+ year credibilities are 2 and 3 times the 1+ year credibility, and if so, only then do you conclude that the variation of an individual insured's chance of an accident is not changing over time. Given this, I'll show the solution that is consistent with the source paper, and then 1 more based on an alternative interpretation of the numbers given in the problem that is not consistent with the source paper (not sure if the graders will allow it).

SOLUTION 1: Source paper approach

We can interpret the given table such that t=1 means 1+ years claim-free, t=2 means 2+ years claim-free, and t=3 means 3+ years claim-free. In other words, if an insured has made it to time 1 without claims, then that insured might also make it to times 2 or 3 or so on without claims. So really, the numbers given are not EXACTLY t years claim-free, but t or more years claim-free. So if we wanted the number of policies with EXACTLY t years claims-free, we can subtract adjacent columns in the given table. For the 0.05 freq group, this would mean 2,500 insured have exactly 0 years claims-free, 2,500 have exactly 1 year claims-free, 1,000 insureds have exactly 2 years claims-free, and 44,000 insureds have 3+ years claims-free.

\# of claims at t = # of insureds claim-free at t × Expected Freq
Freq at t = total # of claims at t / total # of insureds claim-free at t
Credibility = 1 - Relative Freq to t=0 *(note that t=0 is the total of all insureds)*

	# of claims from			
Group	t=0	t=1	t=2	t=3
0.05	2,500	2,375	2,250	2,200
0.10	5,000	4,500	4,300	3,600
0.20	5,000	4,100	3,300	2,800
Total	12,500	10,975	9,850	8,600
Freq	0.1000	0.0971	0.0943	0.0915
Relative Freq to t=0	1	0.9712	0.9426	0.9149
Credibility		0.0288	0.0574	0.0851
Relative Cred to t=0		1	1.996	2.959

Since the relative credibilities for t=2 and t=3 are approximately 2 and 3 times the credibility for t=1, the variation of an individual insured's chances for an accident is not changing over time.

continued on next page

SOLUTION 2: Alternative interpretation, inconsistent with source paper

Assume the numbers given represent # of insureds with EXACTLY t years claims-free.

# of claims at t = # of insureds claim-free at t × Expected Freq

	# of claims from			
Group	t=0	t=1	t=2	t=3
0.05	2,500	2,375	2,250	2,200
0.10	5,000	4,500	4,300	3,600
0.20	5,000	4,100	3,300	2,800
Total	12,500	10,975	9,850	8,600

t+ years Freq = Sum(# of claims from t or more years) / Sum(# of insureds for t or more years)

Total Freq = (12,500 + 10,975 + 9,850 + 8,600) / (125,000 + 113,000 + 104,500 + 94,000) = 0.0960

Credibility = 1 - Relative Freq to total

		Years claims-free		
	Total	1+ years	2+ years	3+ years
Freq	0.0960	0.0945	0.0929	0.0915
Relative Freq to total	1	0.9835	0.9677	0.9525
Credibility		0.0165	0.0323	0.0475
Relative Cred to total		1	1.955	2.875

Since the relative credibilities for 2+ and 3+ are fairly close to 2 and 3 times the credibility for 1+ years claims-free, the variation of an individual insured's chances for an accident is not changing over time.

19. **2017 Exam 8 - Q3 (1.5 points)**

   (a) *I assume all premiums in this question are on-level. We want to use premium as the base instead of car-years if high frequency territories are also high average premium territories, and if territory rates are proper. In this part, we only need to look at the 2nd table given.*

   There isn't enough information to know whether territories are priced correctly, but we can see that high frequency territories are not high premium territories, because territory C has the highest frequency but lowest average premium. As such, I would recommend using earned car-years as the base for frequency.

   (b) *The wording of this question says RELATIVE credibility, so based on the usage of this in the source material, I assume they are asking for the 3+ year credibility divided by the 1+ year credibility. That said, this should have been more explicit, and past exam question have been more explicit that we are asking for credibility relative to 1+ years claim-free.*

   3 or more Freq = (15 + 13.5) / (250 + 100) = 0.081
   1 or more Freq = (15 + 13.5 + 8) / (250 + 100 + 80) = 0.085

   Total Freq = 47 / 500 = 0.094

   3 or more Mod = 0.081 / 0.094 = 0.866
   1 or more Mod = 0.085 / 0.094 = 0.903

   3 or more Cred = 1 - 0.866 = 0.134
   1 or more Cred = 1 - 0.903 = 0.097

   Relative Cred = 0.134 / 0.097 = 1.379

20. **2018 Exam 8 - Q3 (2.75 points)**

(a) $R = \frac{1}{1-e^{-0.05}}$ = 20.504

Mod = (20.504)(0.038) + (1 - 0.038) = 1.741

The premium is given not at B rates, so we need to divide out current relativities to get premium at B rates.

Group	Prem at B Rates ($000,000) = Prem/Curr Factor
A	360
X	180
Y	75
B	200
Total	815

$1.741 = \frac{C/200}{(63000+C)/815}$

$(1.741)\frac{63000+C}{815} = \frac{C}{200}$

0.4272(63,000 + C) = C

2.6918 + 0.4272C = C

C = 47,000

(b) *The wording here is a little odd, as there is no merit rating factor for the A+X category. Instead, there would be an experience mod that would replace the merit rating variable. So I think the question writer meant to ask what the indicated experience mod would be for a risk with 2 or more accident-free years. The answer to that is just the relative frequency for A+X to the total using premium at B rates.*

A+X Frequency = (25 + 18) / (360 + 180) = 0.07963

Total Frequency = (63 + 47) / 815 = 0.13497

Mod = 0.07963 / 0.13497 = 0.59

(c) *Really, because of the wording of this question, there is only 1 main reason, which is that other variables aren't priced properly. The other item from the paper about high frequency territories having high premium is about exposure correlation existing in the first place, not about correcting for it. But I suppose if exposure correlation doesn't exist, then there is nothing to correct.*

- If other rating variables are not priced properly, then EP would not properly account for exposure correlation with those variables.
- If there is no exposure correlation with other rating variables, then there will be no maldistribution issue to correct.

21. **2019 Exam 8 - Q3 (1.75 points)**

(a) $Mod = ZR + (1 - Z)$

R is last year's relative claim frequency of current B ratings to the total class:

$$R = \frac{\text{(\# of claims last year from current 'B' ratings) / (earned car years last year of current 'B' rating insureds)}}{\text{(\# of claims last year from class) / (earned car years last year of insureds in class)}}$$

Since all claims in the class last year came from insureds that now have 'B' ratings, the numerators cancel out:

$$R = \frac{\text{1 / (earned car years last year of current 'B' rating insureds)}}{\text{1 / (earned car years last year of insureds in class)}} = \frac{\text{earned car years last year of insureds in class}}{\text{earned car years last year of current 'B' rating insureds}}$$

The above fraction is now just 1 divided by the portion of insureds that have 'B' ratings. Insureds with 'B' ratings had at least 1 claim last year, so we can get:

$$Pr(X \geq 1) = 1 - Pr(X = 0) = 1 - \left[\frac{(0+r-1)!}{0!(r-1)!} \times p^0 \times (1-p)^r\right] = 1 - [1 \times 1 \times (1-p)^r]$$

$$R = 1/[1 - (1-p)^r] = 1/[1 - (1-p)^{10}]$$

We can get p because we know the expected claim frequency will equal 0.101 and we have the negative binomial formula for the mean.

$$\frac{pr}{1-p} = \frac{10p}{1-p} = 0.101$$

Solve for $p = 0.010$

$$R = 1/[1 - (1 - 0.010)^{10}] = 10.459$$

$$Mod = (0.02)(10.459) + (1 - 0.02) = 1.189$$

(b) *The question wording asks you to describe why a CLASS may have less credibility, but the model solutions in the examiner's report discuss why an INDIVIDUAL RISK within a class might have less credibility. I'll show answers for both, even though credit was only given for the individual risk assumption (even though that's not what the question wording said).*

Solution 1: Talking about individual risk experience rating credibility

Individual risk experience rating credibility is used to distinguish between risks within a class. If the variance between risks in a class is low (i.e., risks within the class are very similar), then experience rating credibility will also be low regardless of the size of the class.

Solution 2: Talking about class credibility

Credibility depends not just on the volume of data, but on the variance of the data. So a class with lots of data could have more variance in loss results than a class with less data, and as such might deserve lower credibility.